

The Coherence of Kant's Synthetic A Priori

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Is there synthetic a priori knowledge? The question can be rephrased as Sellars puts it: "Are there any universal propositions which, though they are not true by virtue of the meaning of their terms, hold of all possible objects of experience?"¹ Kant's project is immense; it is that of answering the question affirmatively in virtue of the "*dependence* between us, our cognitive powers or our language, and the world of our experience."² It would not be misleading to characterise the synthetic a priori as the cornerstone in the epistemic framework of Kant's metaphysics. Central to Kant's argument is the appearance of the synthetic a priori in mathematics. As one may expect of a revolutionary thesis, Kant's doctrine has its detractors – it has done since the publication of the first edition of the *Critique*. In this paper, I will explicate the concept of the synthetic a priori, focusing predominantly on definition and application in mathematics, and will in turn evaluate some of the claims of Kant's critics.

Having the benefit of considering the successes and inadequacies of both rationalism and empiricism,³ Kant concludes that metaphysics has "until now⁴ remained in such a vacillating state of uncertainty and contradictions."⁵ He contends that our metaphysical woes derive from epistemic oversight, and presents the synthetic a priori as a beacon. Some justification is of course required, as this conception (although familiar in current philosophy thanks to Kant) represents two epistemic concepts often thought to be in tension with each other. I will briefly consider first Kant's concept of the a priori, and then turn to the novel distinction between synthetic and analytic judgments.

¹ Sellars, W. (1953). Is There a Synthetic a Priori? *Philosophy of Science*, 20 (2), p123

² Bird, G. (2006). Kant's Analytic Apparatus. In G. Bird (Ed.), *A Companion to Kant*. Oxford: Blackwell, p129, emphasis in the original

³ Scruton (2002): "It was Kant's principal contribution to show that the choice between empiricism and rationalism is unreal, that each philosophy is equally mistaken, and that the only conceivable metaphysics that could commend itself to a reasonable being must be both empiricist and rationalist at once."

⁴ Buroker, J. V. (2006): "It is no exaggeration to say that the precise motivation for Kant's Copernican revolution is his conviction that no predecessor had explicitly recognized this kind of knowledge."

⁵ Kant, I. (2000). *Critique of Pure Reason*. (P. Guyer, & A. Wood, Eds.) Cambridge: Cambridge University Press, B19. (Hereafter referred to as *CPR*.)

Kant argues that we require a priori knowledge because reason, the faculty by which we wish to do metaphysics, desires knowledge of universals and necessity and is not satisfied by knowledge of mere particulars.⁶ A priori knowledge is contrasted with empirical knowledge, in that it does not depend for its justification on experience. Kant proleptically spells out the relationship that a priori knowledge bears to experience when he notes that “although all our cognition commences **with** experience, yet it does not on that account all arise **from** experience.”⁷ Kant here is at pains to demarcate the scope of this dependence: “although all cognition is *temporally* dependent on experience, it does not follow that it is *logically* dependent on it.”⁸ Thus, while it is true that I would never have known that (a) ‘all bachelors are unmarried’ if I had never been born (i.e. had no experiences), the truth of (a) does not *depend logically* on my (or anyone’s) experience.

Let us turn now to the analytic-synthetic distinction. Regarding analytic cognitions, Kant suggests that they are judgments of “clarification,”⁹ that is, they “do nothing more than represent *clearly* and assert as belonging to it, what was already really thought and contained in the given concept.”¹⁰ This squares quite well with the way we frequently describe analytic cognitions as ‘trivial,’ though Kant couches the idea in terms of ‘covert containment of concepts’¹¹ – not a notion typically endorsed today. Kant appears at first glance to be committed to the idea that only statements in subject-predicate form can be analytic. We naturally would wish to repudiate this narrow conception, since we think that (e.g.) conditional statements like ‘if A is longer than B, and B is longer than C, then A is longer than C’ are analytic.¹² However Kant “does occasionally seem to concede that a non-subject-

⁶ C.f. CPR A1, and B4 “necessity and strict universality are therefore secure indications of an a priori cognition.” It will do here to recognise that we typically distinguish the metaphysical role of necessity and the epistemic (justification) role of the a priori. Kant sees these as connected, as Hanna (2004) notes: “Kant’s idea, by sharp contrast, is that semantic, epistemic, and metaphysical considerations cannot be wholly disentangled from one another and are ultimately fused in the more basic and comprehensive notion of cognition or objective mental representation” (p246).

⁷ CPR, B1, emphasis in the original

⁸ Buroker, J. V. (2006), p27

⁹ CPR, B11

¹⁰ Kant, I. (1790). On a Discovery whereby Any New Critique of Pure Reason Is to Be Made Superfluous by an Older One. In H. Allison, & P. Heath (Eds.), *Theoretical Philosophy after 1781* (2002 ed.). Cambridge: Cambridge University Press, 8:228, emphasis in the original

¹¹ In Kant’s words, a statement ‘A is B’ is analytic where the “the predicate B belongs to the subject A as something that is (covertly) contained in this concept A.” (CPR, B10)

¹² Cf. Pap, A. (1950). Logic and the Synthetic A Priori. *Philosophy and Phenomenological Research*, 10 (4), p501

predicate proposition could be analytic”¹³ as in: “that equals added to or subtracted from equals give an equal are analytic propositions.”¹⁴

Synthetic judgments, on the other hand, *extend* our knowledge. They are statements of “amplification” (rather than “clarification”) in that they “add to the concept of the subject a predicate that was not thought in it at all.”¹⁵ Kant here provides an example to evoke the distinction. “If I say: All bodies are extended, then I have not in the least amplified my concept of body, but have merely resolved it, since extension, although not explicitly said of the former concept prior to the judgment, nevertheless was actually thought of it; the judgment is therefore analytic. By contrast, the proposition: Some bodies are heavy, contains something in the predicate that is not actually thought in the general concept of body; it therefore augments my cognition, since it adds something to my concept, and must therefore be called a synthetic judgment.”¹⁶ The suggestion here is that since the concept of extension is (covertly) contained in the general concept of a body, while the concept of weight is not so contained, we correctly call the first cognition analytic and the other synthetic.

Kant adds a facility to his epistemic apparatus here – the ‘principle of contradiction’ – which elicits an alternative assessment of synthetic a priori cognitions. Kant says that “All analytic judgments rest entirely on the principle of contradiction and are by their nature *a priori* cognitions, whether the concepts that serve for their material be empirical or not. For since the predicate of an affirmative analytic judgment is already thought beforehand in the concept of the subject, it cannot be denied of that subject without contradiction.”¹⁷ Synthetic judgments, on the other hand depend on principles in addition to that of contradiction; as Buroker puts it, “In synthetic judgments, there is no identity between the subject and predicate, and so the principle of non-contradiction is not sufficient for determining their truth values.”¹⁸

¹³ Dicker, G. (2004). *Kant's Theory of Knowledge: An Analytical Introduction*. New York: Oxford University Press, p12

¹⁴ *CPR*, A164/B204

¹⁵ *CPR*, B11

¹⁶ Kant, I. (1783). Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science. In H. E. Allison, & P. L. Heath (Eds.), *Theoretical Philosophy After 1781* (G. Hatfield, Trans., 2002 ed.). Cambridge: Cambridge University Press, 4:266-267 (Hereafter referred to as *Prolegomena*.)

¹⁷ *Prolegomena* 4:267

¹⁸ Buroker, J. V. (2006), p29

There is, however, a difficulty concealed here. Graham Bird points out helpfully that a little reformulation will be of assistance at this point in order to accommodate “a conflict between applying the classifications to ‘judgments’... or to ‘true judgments.’”¹⁹ We wish to say that a judgment like “some humans are not mammals” is *an analytic judgment*, even though its denial does not yield a contradiction but an analytic truth. This matter can be easily ironed out by suggesting rather that “A judgment is analytic (synthetic) if and only if either it or its denial (neither it nor its denial) is a contradiction.”²⁰

With this tool in hand, let us turn to Kant’s famed proposal, which “appears to have completely escaped the observations of analysts of human reason,”²¹ namely, that mathematical truths are synthetic and a priori. If we can come up with examples which are incontrovertibly both synthetic and a priori, it will succeed in establishing the coherence of the synthetic a priori. Kant urges us to consider the sum ‘ $7 + 5 = 12$,’ and to apply to it the definitions he has expounded. It is plain that it is a priori, for it in no way depends on experience for its justification.²² I think Kant is successful too in showing that the sum, when considered in terms of ‘covert conceptual containment’ (CCC), comes out synthetic; that is, “upon closer inspection, one finds that the concept of the sum of 7 and 5 contains nothing further than the unification of the two numbers into one, through which by no means is thought what this single number may be that combines the two.”²³ Kant notes correctly that this “can be seen all the more plainly in the case of somewhat larger numbers:”²⁴ it is a fortiori clear that (e.g.) the concept of ‘ $26589 + 56421$ ’ is not thought to contain the concept of ‘ 83010 ’.

But what of the second criterion, the ‘contradiction test’ (CT)? If we apply CT to the aforementioned sum, we are placed in a difficult position. It seems that supposing that the sum turned out otherwise (say, $7 + 5 = 13$) renders the equation incomprehensible, maybe as incomprehensible as trying to imagine a married bachelor. Kant at this point urges upon us a principle to resolve our confusion, similar to that distinction between truths

¹⁹ Bird, G. (2006), p127

²⁰ Bird, G. (2006), p128

²¹ *Prolegomena* 4:268

²² *Prolegomena* 4:268, “Properly mathematical propositions are always *a priori* and not empirical judgments, because they carry necessity with them, which cannot be taken from experience. But if this will not be granted me, very well, I will restrict my proposition to *pure mathematics*, the concept of which already conveys that it contains not empirical but only pure cognition *a priori*.”

²³ *Prolegomena* 4:268, CPR B15

²⁴ *Prolegomena* 4:269 CPR B16

that *arise in* and those that *depend on* experience. In considering CT, Kant suggests that we distinguish those inferences that *merely accord with* CT and those which can be *derived through* CT. The inferences of mathematics, according to Kant, certainly “all proceed in accordance with the principle of contradiction (which, by nature, is required of any apodictic certainty).”²⁵ Yet Kant argues that mathematics falls on the synthetic side of this division: “for a synthetic proposition can of course be discerned in accordance with the principle of contradiction, but only insofar as another synthetic proposition is presupposed from which the first can be deduced, never however in itself.”²⁶

The last point has come under some attack, notably from Ayer, since Kant does not ever show precisely how CT is to be applied successfully to arithmetic, but focuses rather on conceptual containment (CCC). Ayer suggests that these two tests can produce conflicting results – that is, e.g., something could turn out to be synthetic a priori according to CCC but analytic according to CT. Ayer says “But, in fact, a proposition which is synthetic according to the former criterion may very well be analytic according to the latter... one can think of the sum of seven and five without necessarily thinking of twelve, [but] it by no means follows that the proposition “ $7 + 5 = 12$ ” can be denied without self-contradiction.”²⁷

I think, however, that Ayer has misinterpreted Kant’s distinction, and that his comments incline towards a conflation of judgments which *accord with the principle of contradiction* and those which are *derived through the principle of contradiction*. Furthermore, I wish to argue that Kant could contend successfully that CCC entails CT.²⁸ I will use examples to illustrate my contentions.

It is plain that the denial of an analytic truth like ‘all bachelors are unmarried’ results in a contradiction. Why? Because we have the relevantly conflicting concepts immediately in mind. That is, ‘some unmarried men are married men’ is contradictory *at face value*, because the concepts stand in perfect opposition to each other: we have [U and not-U], generating a contradiction. However, in considering the sum ‘ $26589 + 56421$,’ we are simply not presented with the concept ‘83010’, covertly or otherwise. But if we are not presented with such a concept, then *though it is indeed false to deny the equation, it is just not a contradiction*. It is correct to advance, then

²⁵ *Prolegomena* 4:268, CPR B14

²⁶ *Prolegomena* 4:268, CPR B14

²⁷ Ayer, A. (1952). *Language, Truth, and Logic*. New York: Dover Publications, p78

²⁸ Bird, G. (2006), p128, also adheres to this formulation.

that genuine contradiction arises only where the concepts in the cognition have opposing truth values; in other words, we can conclude that a cognition which comes out as synthetic a priori on CCC will necessarily come out as synthetic a priori on CT.

As alluded to earlier, I have attempted to satisfy the question ‘Is Kant’s conception of synthetic a priori propositions defensible?’ by showing that we in fact have instances of such propositions – what is actual is of course possible. So, by way of conclusion, I will evaluate two additional mathematical examples which count as synthetic a priori cognitions *on Kant’s terms*. The first is Kant’s, proposing that “the straight line between two points is the shortest”²⁹ is a synthetic a priori proposition. How does it fare with CCC and CT? Well, on CCC it is clear that the concept of a straight line does not contain anything “of magnitude,”³⁰ that is, we cannot by considering the concepts alone derive the concept of ‘the shortest distance’. And by this token – since wholly distinct concepts figure in the subject and predicate terms – we see that the denial of the statement does not result in contradiction. So on CT it is similarly synthetic a priori.

A neat example from the proliferation of literature on the subject in the fifties comes from Langford, who proposes that “‘any cube has twelve edges’ is both synthetic and a priori.”³¹ Without labouring the point, it should be clear that the concept “solid whose faces were squares”³² does not contain the concept of ‘having twelve edges.’ And similarly (indeed consequently), while patently *false* to be sure, it is not *contradictory* to suppose otherwise.

While Kant’s project proceeds into murkier territories hereafter, I contend that his groundwork – that of establishing the coherence of synthetic a priori cognitions – provides a solid foundation for setting metaphysics “along the secure course of a science.”³³

²⁹ *Prolegomena* 4:269, CPR B16

³⁰ *Prolegomena* 4:269

³¹ Langford, C. (1949). A Proof That Synthetic A Priori Propositions Exist. *The Journal of Philosophy*, 46 (1), p21

³² Langford, C. (1949), p22

³³ CPR, Bxiv

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